Unification without Unification

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The logarithmic running of the gauge couplings α_1, α_2 and α_3, indicates that they may unify at some scale $M_{GUT} \sim 10^{16}$GeV. This is often taken to imply that the standard model gauge group is embedded into some larger simple group in which quarks and leptons are placed in the same multiplet. These models have generic features, such as proton decay, and generic problems, namely the splitting of the Higgs doublet and triplet. Inspired by the recent discussion of dimensional deconstruction, we propose an interesting alternative: we postulate a strongly coupled $SU(3) \otimes SU(2) \otimes U(1)$, which is not the remnant of a GUT, and is Higgsed with a weakly coupled $SU(3) \otimes SU(2) \otimes U(1)$, which is the remnant of a GUT, or with a GUT group directly, into the diagonal subgroup. In this "collapsed GUT" mechanism, unification of coupling constants in the low energy theory is expected, but proton decay and the doublet/triplet splitting problem are entirely absent.

I. INTRODUCTION

The standard model, consisting of the gauge group $SU(3)\otimes SU(2)\otimes U(1)$ broken at the weak scale to $SU(3)\otimes U(1)$, has been extremely successful. Nonetheless, it has a number distasteful features, which has prompted a great deal of study into the possibilities of physics beyond the standard model.

It is exceptionally notable that two significant proposals of physics beyond the standard model seem quite complementary, namely Grand Unified Theories (GUTs) and supersymmetry (SUSY). As precision measurements on the gauge couplings α_1, α_2 and α_3 have improved, it has become increasingly clear that the original GUTs - without supersymmetry [1,2] - do not agree with the measured value of $\sin^2\theta_W$. However, GUTs with supersymmetry [3,4] seem to agree quite well with precision data [5]. Indeed, this is often considered a great success of supersymmetry.

A generic feature of GUTs is the instability of the proton, which occurs dominantly through either X and Y boson mediated dimension six operators, or through triplet Higgsino mediated dimension five operators. The triplet Higgs mass is a free parameter, but often the dimension five operators are the dominant source of proton decay [7,8]. Together with precision measurements on the gauge couplings, this has been used to exclude various models of grand unification [9,10].

While GUTs are theoretically appealing, they are not without problems. Probably the greatest is the doublet/triplet splitting problem. Because of the larger gauge group, the Higgs comes with a triplet partner, whose mass must in general be near the GUT scale. In "minimal" $SU(5)$, the GUT is broken by a field Σ, which transforms as a **24** under $SU(5)$. The bare mass superpotential term $mH\overline{H}$ is then tuned against a $\Sigma H\overline{H}$ term to give the doublet a small mass, while leaving the triplet with a mass $O(M_{GUT})$.

A number of solutions exist for this, including the missing partner mechanism [11,12], the Higgs as a pseudo-goldstone boson [13,14], and others. We shall not discuss the merits and drawbacks of each here, but clearly some solution is in order.

A. Grand Unification?

Before we continue further, let us reexamine what the evidence is for grand unification. Given the particle content of the standard model, we can study the renormalization group evolution of the gauge couplings from the weak scale to higher energy scales. We then extrapolate over *fourteen decades of energy*, assuming nothing but MSSM fields enter into the RGEs. This evidence for grand unification is quite indirect.

At the same time, there is also indirect evidence against grand unification. There is the absence of any proton decay signal, but additionally, the expected relations between m_e, m_μ, m_d and m_s fail by an order of magnitude. Given the additional complexities that are necessary to solve the doublet/triplet splitting problem, it is perhaps worthwhile to question whether we *must* read the gauge coupling unification as an indication of the standard model being embedded in a unified group. Put simply: can we understand coupling unification without a conventional GUT?

Various proposals have put forth to this end. For instance, in [15] it was proposed that coupling constant unification could occur in a strongly coupled theory. Other possibilities include using a different group structure [16], or with unification at the string scale [17], without a grand unified group.

In this letter, we will see how an enlarged gauge symmetry can naturally give gauge coupling unification without having a grand unified theory in the conventional sense. The outline is as follows: in section II we shall discuss how Higgsing a strongly coupled sector into a weakly coupled remnant of a grand unified group gives the appearance of unification, what we lose from such a scenario, and how such breaking might occur. In section

III we comment on such a scenario in theories with TeV -sized extra dimensions and gauge coupling unification at a low scale, $O(10\text{TeV})$.

II. UNIFICATION WITHOUT UNIFICATION

We begin by considering the well known case of two copies of a single gauge group G, with couplings g_1 and g_2. If the theory is Higgsed down to the diagonal subgroup, the gauge coupling of the resulting massless gauge boson is given by the well-known formula

$$\frac{1}{g_{eff}^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}. \tag{1}$$

The situation we shall be most interested in is the case in which g_1 is small and g_2 is large. In this case, $g_{eff} \approx g_1$.

With this simple fact in hand, we can consider the following scenario. Consider a model in which the gauge symmetry is $G_W \otimes G_S$. As before, G_W will be weakly coupled, while G_S will be strongly coupled at the GUT scale. G_W will be some semi-simple group which contains $SU(3)\otimes SU(2)\otimes U(1)$ as a subgroup, but not the groups under which quarks and leptons are charged. Instead, let us take G_S to also contain a copy of $SU(3)\otimes SU(2)\otimes U(1)$ - although it may be larger - under which quarks and leptons are charged.

At some scale $M_D \leq M_{GUT}$, we assume some additional dynamics acts to Higgs the $G_W \otimes G_S$ down to its diagonal $(SU(3)\otimes SU(2)\otimes U(1))^2$ subgroup. This may occur simultaneously with the GUT breaking or at a lower scale. For the practical purpose of achieving gauge coupling unification, it is generally best that these happen simultaneously. However, since $g_S \gg g_W$, then as before we have $g_{eff} \approx g_W$, except now *the standard model fields are charged under this group!*

Let us study the RG evolution of the gauge couplings to extrapolate to low energies. At the GUT scale, the 3-2-1 gauge couplings are strong, while the $SU(5)$ coupling is weak. Since we are assuming that the only non-$SU(5)$ complete multiplets lie in the standard model sector, the gauge couplings run from the GUT scale as

$$\begin{aligned} \alpha_i^{-1}(\mu) &= \alpha_i^{-1}(M_{GUT}) && (2)\\ &\quad +(\text{MSSM running}) + (\text{universal running}), && (3)\\ \alpha_5^{-1}(\mu) &= \alpha_5^{-1}(M_{GUT}) && (4)\\ &\quad +(\text{universal running}). && (5)\end{aligned}$$

When we Higgs to the diagonal subgroup, leaving only 3-2-1, the gauge couplings of the remaing gauge group are just the sum of these. Explicitly, at a scale $\mu < M_D$, we have

$$\begin{aligned} \alpha_{(LE)i}^{-1}(\mu) &= \alpha_i^{-1}(M_{GUT}) + \alpha_5(M_{GUT}) && (6)\\ &\quad + \frac{b_{i,MSSM}}{2\pi}\log(M_{GUT}/\mu) && (7)\\ &\quad +(\text{SU(5) universal}), && (8)\end{aligned}$$

where we use LE (low energy) to distinguish the gauge coupling of the remaining massless group after Higgsing from the corresponding coupling above M_D. Note the MSSM running is independent of the scale of breaking.

At the weak scale, this will be indistinguishable from an ordinary GUT up to corrections arising from the $\alpha_i^{-1}(M_{GUT})$. We would like these corrections to be comparable to those expected in ordinary GUT theories, requiring $\alpha_i(M_{GUT}) \geq 1$, so the theory is somewhat strongly coupled, but still perturbative.

We now have a remarkable situation: at low energies, the gauge couplings are consistent with being embedded within a grand unified group. However, there is no proton decay from X and Y exchange as quarks and leptons are not charged under the GUT. There is no proton decay from the Higgs triplet because there is no Higgs triplet in the theory. Moreover, the Yukawas will not obey any GUT relationships.

This scenario is reminiscent of a model of doublet/triplet splitting proposed in [18], in which the gauge group $SU(5)\otimes SU(3)\otimes U(1)$ was postulated to give the Higgs triplet a large mass. The differences are profound, however: there, the standard model really was embedded into a unified group. Here it is not.

A. Collapsed GUTs and Dimensional Deconstruction

The recent surge of interest in “Moose” [19] or “Quiver” [20] models has been spurred by the realization that such theories can serve as UV completions of higher dimensional gauge theories [21,22], and provide useful features of the higher dimensional theories without the associated UV problems.

This mechanism also has a clear analog in higher dimensions: that of the GUT broken by orbifold boundary conditions [23]. The idea of breaking grand unified theories by Wilson lines has existed for a great while [24–26], and has recently seen a resurgence of its application in realistic model building [27–30]. In these models, at a specific point in the fifth dimension, the SU(5) gauge transformation vanishes. We can identify this point as the site on a Moose diagram where we merely have 3-2-1 gauge group. While a direct deconstruction would in general have multiple SU(5) sites, while we have but one, the connection is clear. A more general discussion of deconstruction would be interesting [31].

B. What have we lost?

Grand unified theories do have many desirable features [32]. Now that the standard model is not grand unified,

we lose many of these, but not as much as might be expected.

For instance, we have the charge assignments of the MSSM chiral matter fields. Since any underlying theory can only should only generate consistent quantum theories, we can still understand this through anomaly cancellation. Nonetheless, ther overall normalization of hypercharge is still undetermined. One might make assumptions that the fundamental string theory naturally gives the proper normalization, but this is not necessary. A perhaps more natural assumption is that 3-2-1 is contained in a nonabelian gauge group, in which the charge assignments are automatic. Two obvious examples would be $SU(3)^3$ [33] and $SU(4) \otimes SU(2) \otimes SU(2)$ [34].

Although there is no right handed neutrino in this model, we still expect heavy states at M_{GUT}, so if lepton number is broken there, we still understand neutrino masses. In any event, there are a number of ways to understand neutrino masses in supersymmetric theories [35–38].

Additional symmetries are easily added to the theory, such as lepton number, $B - L$, and Froggatt-Nielsen symmetries. The unification of bottom and tau Yukawas is an important success in certain regions of parameter space, but it is not a generic success of the MSSM [39].

In conclusion, while these are many successes which arise from grand unification, for the most part they can be included in this framework.

C. Breaking to the diagonal subgroup

It is simple to break $SU(5) \otimes SU(3) \otimes SU(2) \otimes U(1)$ to the diagonal subgroup. An explicit linear sigma model was given in [40]. One could also imagine using strong dynamics if fields charged under $SU(5)$ and under $SU(3) \otimes SU(2) \otimes U(1)$ condensed, breaking to the diagonal. Of course, this has swept various questions into the guise of strong dynamics. For instace, we have no clear understanding of why it is broken precisely to the diagonal subgroup, rather than some other subgroup. However, for a non-SUSY GUT model, these are certainly necessary questions, and warrant the development of a realistic model.

In general, the $3-2-1$ sector should be strongly coupled at or near the GUT scale. Absent additional matter content, $SU(3)$ remains asymptotically free. Thus, for M_D to lie significantly below the GUT scale, additional multiplets are needed. However, the matter used to break to the diagonal subgroup can serve this purpose, for example the fields in [40]. We also assume the fields in the breaking sector should only contain fields which appear in complete $SU(5)$ multiplets in order not to spoil the quantitative success of grand unification. One can imagine including fields in which there were incomplete multiplets, but this can only be addressed within a specific model.

Finally, we should make one comment regarding scales: without additional matter content, $SU(2)_S \otimes U(1)_S$ is infrared free, while $SU(3)$ is asymptotically free. Thus, to have all couplings be simultaneously strong at the GUT scale is a significant constraint on the theory. In particular, a strong U(1) at the GUT scale will have a Landau pole before the Planck scale, so it cannot be considered truly fundamental unless the string scale is lowered. We should note, however, that these problems are irrelevant once we have embedded the U(1) into a non-Abelian group. Since this is already motivated by hypercharge assignment, the asymptotic freedom of the model is not a great concern.

III. TEV SCALES AND PHENOMENOLOGY

Does this scenario have any unique phenomenology? Outside of the absence of proton decay and the non-unification of Yukawas, there is no obvious signal. However, there is a great deal of dependence on the scale at which the strong and weak groups are Higgsed to the diagonal subgroup. If this occurs at a scale significantly below M_{GUT}, there could be noticeable threshold effects if any multiplets are split. Of course, we already expect some level of non-universal effects from the $3-2-1$ gauge couplings themselves.

Other possibilities arise when we add additional structure. In supersymmetric theories, the RG contribution to the soft scalar masses should be modified above M_D, and so could be incompatible with mSUGRA depending on the size of the effect. Moreover, because the gauginos will mix, their masses need not unify.

We now have the possibility of adding other gauge groups, such as $U(1)_B$ which are incompatible with $SU(5)$, for instance, so long as it is made anomaly free. There are no doubt other interesting extensions.

There is another exciting possibility, however, which is that the Higgsing to the diagonal subgroup occurs near the TeV scale. In such a scenario, at upcoming colliders, we would expect to see new $3-2-1$ gauge bosons with strong couplings to quarks and leptons. Such a possibility is especially attractive in models with TeV scale GUTs proposed in ref. [41,42]. Quantitatively, we must address such a possibility. If there are TeV scale GUTs, then quantitative unification is not as precise in SUSY GUTs, so the constraint on the strength of the gauge couplings at the GUT scale may be weaker.

IV. CONCLUSIONS

The apparent unification of gauge couplings has given us motivation for considering grand unified theories.

However, the absence of proton decay, the non-unification of Yukawas and the doublet/triplet splitting problem warrant consideration of other possibilities. At the most minimal level, we only have the unification of coupling constants, so we need also ask whether that alone can be explained without conventional grand unification.

Here we have demonstrated a scenario in which this can happen naturally. By Higgsing a weakly coupled $SU(3) \otimes SU(2) \otimes U(1)$ arising out of a unified group with a strongly coupled copy under which quarks and leptons transform, it will be in many cases indistinguishable from a unified theory, up to the absence of proton decay and constraints on the Yukawas. At present, we have merely described a mechanism, involving unknown dynamics. The development of a complete model is an worthwhile task.

Such a scenario might be interesting to consider if Higgsed to the diagonal group at the TeV scale, or when embedded into TeV scale GUTs. There is a wealth of phenomenology to be undertaken.

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